

# ***SimCalc: Democratizing the Mathematics of Change***

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## **SimCalc: Democratizing the Mathematics of Change**

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### **Why use technology?**

For more than 30 years, I've been interested in mathematical representations—by which I mean any type of mathematical notation, such as numbers, equations, graphs, or even algebra, as a representation system—and the power of different mathematical representations to expose different aspects of complex mathematical ideas. The mathematics we've inherited was developed in a very different world, defined by static, inert media such as clay tablets, paper and pens, and the printed page. These media constrained mathematics and limited the forms of mathematical representations. Further, mathematics historically has been created by and for a very small intellectual elite. But those two constraints no longer apply. We now have dynamic, interactive media available through the use of technology, and this is changing the mathematics that is possible and needed. It is also making mathematics learnable in new ways. And we now expect a much broader segment of the population to learn serious mathematical ideas. I began working with learning technology in mathematics—and helped created the SimCalc Project—in response to these two fundamental changes in mathematics in the latter part of the 20<sup>th</sup> century: 1) a change in the nature of the media in which we can build mathematical representations, and 2) a change in the population that we expect to learn and use serious mathematics. I think it's important for folks to consider these issues in fairly broad historical terms, because it helps us avoid such arguments as, “Should freshman take ‘X’ course,” or “Should we include rationalizing denominators in our pre-calculus curriculum.”

There's another distinction worth making, and that is calculus as a web of ideas, skills, and techniques, versus calculus the institution—the collection of courses, prerequisites, textbooks, and expectations for the 5-10 percent of all students who study calculus.

The SimCalc team came together to democratize access to the core ideas of calculus, the mathematics of change and variation. The main ideas of calculus—rates, variation, mean values, approximations, accumulation, and limits—are relevant to everyday life and especially to science, engineering, and business. But approximately 90 percent of students never learn calculus, having been discouraged by the long list of prerequisite math courses they must take first. SimCalc motion simulations, visualization tools, and innovative teaching strategies provide multiple ways of learning mathematics that are rooted in students' experience.

### **The strategy**

When computational media really became more available by the early 1980s, I immediately started to work with LOGO and other programming languages to help students learn mathematics. Later on, in the mid-1980s, I joined the Educational Technology Center at Harvard University, where we began to experiment more richly with, for example, making the ideas of ratio and proportion more learnable by using what

I call “cybernetic manipulatives.” These are objects that behave like visual objects on a computer screen, but you move them around with a mouse to aggregate them, connect them, or stretch them, for example: the computer provides new capabilities in this process, because the actual physical medium has certain constraints that the mathematical ideas can go beyond.

One example of this is to take a collection of loose objects of a given size, bring them together visually on a screen into a chunk, and declare that to be a new kind of unit. Then you make the new unit small again and take collections of these new units, bunch them together, and create yet another level of unit. This gives you the opportunity to create and manipulate objects that are more mathematical than the physical ones are, and you’re able to create linkages between representations. In this case, these actions can be linked to number sentences.

Since the mid 1980s, we linked tables, graphs, and equations. Hot-linked representations like this are extremely important—you can see virtually instantly the results of changes in one representation across other representations. The most common example is where you take a coordinate graph of a function and stretch it or move it around and see how its equation changes in real time.

At that time, we weren’t really working for large-scale implementation; we were basically experimenting with what computers could do in terms of improving the ability to learn that particular important set of ideas: ratios and proportional reasoning. We had some fairly strong results there, in which 50 sixth-graders were able to do things that the experts said they shouldn’t be able to do for several more years.

Then, in the late 1980s, I had this notion of using motion simulations to make calculus ideas more learnable, in particular using things like velocity and position graphs hot-linked to motion simulations. We wrote a proposal to the National Science Foundation that was funded from 1993 to 1996 and that became the SimCalc Project. The goal was to democratize access to the key ideas involved in calculus and to do so in ways that don’t depend on heavy algebraic prerequisites. These prerequisites weed out 90 percent of the population from the bigger set of ideas beyond calculus, the mathematics of change and variation—it’s a broader pile of mathematics than just calculus.

We went on to do prototyping and testing with students—what’s learnable and so on, building the software and curriculum around it to try to make some of these key ideas learnable by mainstream students. We then received a follow-up grant for another three years, from 1996 to 1999, with one of the main goals being to integrate some of these approaches into mainstream curricula. The other major issue was to make it learnable by instructors, because those are the people who have to implement it. To make this innovation systemic, we had to come up with teacher training materials and new assessments, among other things. Currently, we are working on expanding these ideas in the context of wireless classroom connectivity, mixing hand-helds and larger computers.

### **The courses**

Our initial target population was middle school students, and the materials we developed actually had different versions that were used at any level from grade six to grade 14. Some versions are for middle school students and teachers, some are for high school students, and some versions are for pre-service and in-service high school teachers.

At the university level, use by undergraduates to this point has been in teacher education classes, with some versions used as a front-end for either Pre-Calculus or engineering-type Calculus at my university. But in the future the software will be much more heavily used in Pre-Calculus and Calculus courses. I find a way to use it in most of my classes. It is also used in an AP Calculus course.

### **The learning technology**

*SimCalc*: SimCalc is an innovative combination of software simulations and curricular activities for students beginning in grade six and continuing through college calculus. The software, *MathWorlds*, presents students with animated worlds in which actors—"The Clown and the Dude," "Baby Duck and Mother Duck" and any of a larger set of animations (including simple dots that can replace the animations at any time)—move according to linked graphs. *MathWorlds* also hot-links position, velocity, and acceleration graphs: when students make a change in one graph, they see the corresponding change in the other two graphs. In effect, we build differentiation and integration into the base representation system. This is analogous to how the base-10 placeholder representation system builds the hierarchical exponential structure of numbers into the representation system.

*MathWorlds* is also linked to Microcomputer-Based Lab (MBL) software, which allows students to enter their own body motions into *MathWorlds*. Their motions then become the motions of an "actor" on the *MathWorlds*' screen, allowing the students to explore the mathematical properties of their own motions.

Another component, the *Line Becomes Motion (LBM)* apparatus developed by TERC (<http://www.terc.edu/>), presents a graph that can drive a physical device, for example two cars moving on a track. TERC's *Bouncing Cart* lets students explore a chaotic system by controlling the amplitude and frequency of the piston action that drives the cart.

SimCalc *MathWorlds* is available free of charge for both the PC and Macintosh platforms at: <http://www.simcalc.umassd.edu/>.

*TI-83 Plus Flash* software: This is the first of a series of dynamic visualization tools, for handheld devices and eventually for bigger computers, that teachers can use for teaching algebra. The *TI-83 Plus Flash* software is for use with Texas Instruments graphing calculators and is available for a fee from: <http://epsstore.ti.com/webs/Home.asp>. You can download the shareware version (Calculator MathWorlds Shareware version 1) of the software, a user guide, and an interactive guided tour of Calculator MathWorlds at: <http://www.simcalc.com/products.htm>. Recently, we have developed the ability to transfer documents between the graphing calculator and computer versions of the software, along with parallel curricula. Hence, it doesn't matter which hardware platform you have, you can use either—or both!

### **The funding**

The SimCalc project received two \$3 million grants from the National Science Foundation to develop, prototype, test, and integrate the SimCalc software into curricula. The SimCalc Team and its partners include dedicated educators, developers, and researchers at: University of Massachusetts-Dartmouth (Jim Kaput, Principal

Investigator); TERC; Rutgers University-Newark; San Diego State University; University of Texas-Austin; Syracuse University, and SRI International, Inc.

### **The results**

Generally, the students like the SimCalc software quite a lot. They feel that they're seeing mathematics and experiencing it in a new way. Student learning data are quite impressive, particularly among disadvantaged younger students who would otherwise rarely be given the opportunity to encounter these ideas.

Our next step, the SimCalc 3 project, has just received funding for three more years to exploit networked hand-held devices and computers in mathematics classrooms. There are two new innovations available that we will explore. One is the ability to run serious software on hand-held devices like graphing calculators, Palm Pilots, and such. The other innovation is the ability to network these devices to each other and to desktop-type computers on a wireless basis. Our new project is to figure out how to use this new level of connectivity that links a single computer for the teacher with hand-held devices for the students. This could range from trivial tasks such as downloading and uploading homework, to allowing each student to upload his or her own function that controls a character in a marching parade, for example. There are all sorts of opportunities for interesting activities using hand-held devices linked to a teacher's workstation. We have two major commercial partners in this project: Texas Instruments and Palm.

Part of what we're interested in getting a better understanding of the different kinds of activity structures that are possible and appropriate in this kind of universe that will actually contribute to learning mathematics. For example, if we're all contributing to and working on a shared mathematical object, or even participating in the same mathematical object, the result could be a new way to integrate the social structure of the classroom with the mathematical structures that we want students to be learning.

If you have any questions, you can contact me at: [jkaput@umassd.edu](mailto:jkaput@umassd.edu).

LINKS	
<i>SimCalc:</i>	<a href="http://www.simcalc.umassd.edu/">http://www.simcalc.umassd.edu/</a>
<i>MathWorlds (downloads):</i>	<a href="http://www.simcalc.umassd.edu/NewWebsite/technology.html">http://www.simcalc.umassd.edu/NewWebsite/technology.html</a>
<i>Line Becomes Motion (LBM):</i>	<a href="http://www.simcalc.umassd.edu/NewWebsite/sc2lbm.html">http://www.simcalc.umassd.edu/NewWebsite/sc2lbm.html</a>
<i>Bouncing Cart:</i>	<a href="http://www.simcalc.umassd.edu/NewWebsite/cart.html">http://www.simcalc.umassd.edu/NewWebsite/cart.html</a>
<i>TERC:</i>	<a href="http://www.terc.edu/">http://www.terc.edu/</a>
<i>TI-83 Plus:</i>	<a href="http://education.ti.com/product/tech/83p/features/features.html">http://education.ti.com/product/tech/83p/features/features.html</a>