How can mathematics educators better understand the early stages of algebraic reasoning in children?

How can these stages be developed in the context of elementary school mathematics in ways that reach many schools and are sustainable?

Although elementary teachers are a critical part of algebra reform, they typically have little experience with the rich and connected kinds of algebra that need to become the norm in their instruction. James Kaput (University of Massachusetts-Dartmouth) and colleagues at WCER’s National Center for Improving Student Learning and Achievement (NCISLA) in mathematics and science are developing ways to enhance teachers’ understanding of the kinds of algebraic thinking that needs to occur in elementary school mathematics. Their research is funded by the Office of Educational Research and Improvement, U.S. Department of Education.

Kaput pursues an “algebrafication” strategy that embodies three dimensions of teacher-based classroom change:

- using available instructional materials to build opportunities for algebraic reasoning, especially generalization and progressive formalizing of those generalizations
- building teachers’ “algebra eyes and ears” so they can recognize opportunities for generalization—and systematic expression of that generality—and then act on these as they occur
- creating classroom practice and culture to support students’ purposeful conjecture and argument, so that algebra opportunities occur frequently and are viable when they occur.
Improving communication

As the stories in this issue of Highlights took shape, the theme of effective communication emerged. Although two of the stories treat mathematics education and two treat English/language arts education, they share effective communication as an important focus.

Two stories result from recent work at WCER’s National Center for Improving Student Learning and Achievement in mathematics and science. The work of Jim Kaput and colleagues found examples of greater student achievement in mathematics when students worked in pairs than when working individually. The work of Tom Romberg and colleagues shows how teachers’ early, formative assessment of student progress enhances classroom communication and student achievement.

Recent work at WCER’s Center for English Learning and Achievement complements these stories. The work of Marty Nystrand and colleagues shows how teachers can create conditions under which students compose more effective argumentative essays, and the work of Jane Zuengler and colleagues suggests how to improve communication with students who are not native speakers of English.

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Andy Porter

Algebraification is a deliberate capacity-building strategy that enables teachers to expand their curricular and pedagogical capacities and, in so doing, also deepen their content understanding of elementary mathematics. Teachers learn how to use their own instructional resource base and their own classrooms as venues for structuring algebra opportunities in elementary school mathematics.

While working with 15 teachers from six elementary schools, Kaput and colleague Maria Blanton collected written and oral reports and sample student work. They then focused on the classroom practice of one third-grade teacher, “Jan,” who was creating the kind of algebraified instructional context that reflected the researchers’ work with teachers. Compelling evidence was gathered during the past few months to support Kaput’s “algebraification” strategy and the specific forms that it can take “on the ground.”

“In Jan’s classroom we see evidence of student achievement that validates our professional development activities with teachers,” says Kaput. “We have also used our case study of Jan’s practice to identify what it means for a teacher’s classroom practice to be ‘algebraified’ in a way that is generative and self-sustaining.”

The theme of “learning with understanding” guides this project. “The most important feature of learning with understanding is that it is generative,” says Kaput. “When students or teachers acquire knowledge with understanding, they can apply the knowledge to learn new topics and solve new and unfamiliar problems. When students or teachers do not understand, they perceive each topic as an isolated skill. They cannot apply their skills to solve problems not explicitly taught to them, nor extend their learning to new topics.”

This past spring the team administered a set of fourth-grade MCAS (Massachusetts Comprehensive Assessment System) exams to Jan’s class of 14 third-grade students, who completed the exam individually, then again with partners in seven dyads. Combined results show that these third-grade students performed significantly above the “proficient” level on 33 percent of the items and were at or above the “needs improvement” level for 67 percent of the items. These third graders from a low SES neighborhood outperformed district and state means for fourth graders.

Students’ individual performances, although varied, also support the promising results indicated by the item analysis. In particular, out of 14 students, 1 student performed at the “advanced” level, 3 students performed at the “proficient” level, and 6 students performed at the “needs improvement” level. Only 4 students ranked lower.

Paired-student results on the assessment were quite exceptional, and reveal perhaps more adequately than individual scores what these students really understand, because they were able to externalize their thinking and bring their linguistic resources into play. Out of the seven partner groups, four groups performed at the “advanced” level, two groups performed at the “proficient” level, and one group performed at the level “needs improvement.” “The fact that this was a fourth-grade assessment makes these results particularly compelling,” Kaput says.

This study of Jan’s teaching revealed characteristics of teacher change in the context of an algebraified classroom practice, specifically

(a) an ability to generalize an arithmetic activity to an algebraic activity,
(b) the use of algebraic conversations in the classroom on a regular basis,
(c) the use of algebraic themes over significant periods of time, and
(d) the use of multiple algebraic processes in a single mathematical task.

For more information see the article by Blanton and Kaput, “Characterizing generative and self-sustaining teacher change in a classroom practice that promotes students’ algebraic thinking,” available through the NCISLA web site at www.wcer.wisc.edu/NCISLA.
Fundamental changes in teachers’ instruction can result from showing them the value of conducting formative assessments. Formative assessment occurs when teachers give students immediate, contextualized feedback during the learning process, so that they can learn better. It is diagnostic and can be informal. Summative assessment, on the other hand, usually occurs at the end of a course. Most standardized tests are summative.

Formative assessment is almost nonexistent in U.S. classrooms, but it has been shown to help teachers with instructional decision making, interpreting students’ written and verbal responses, and eliciting or responding to student ideas during the course of instruction. WCER researchers are developing and testing a program of professional development that seeks to bring about these changes.

Although standardized tests measure end-result student achievement, they do not measure formative gains or strength of reasoning, says UW–Madison Education Professor Thomas A. Romberg, a codirector of this study.

“Standardized tests are valid in a limited way, generally measuring students’ rote retention of formulae or superficial understanding of number, algebra, or geometry,” Romberg says. As such, they do not measure students’ depth of reasoning or potential achievement. Neither can they effectively be used to inform day-to-day instruction or to assess the immediate or short-term needs of the class as a whole or of the individual student.

Consequently, Romberg and colleagues from the Freudenthal Institute, The Netherlands, advocate and provide professional development in formative classroom assessment. Participating teachers receive on-site, classroom- and teacher-specific support from the staff developers, lead teachers, and researchers.

**How students benefit**

Formative assessments help students learn with understanding critical concepts in mathematics, says Jan deLange, a longtime collaborator with Romberg and director of the Freudenthal Institute. deLange has found that classroom teachers initially have limited understanding of reform assessment practices. But when teachers used formative assessment as part of a professional development program, their students’ achievement improved.

In the process of professional development workshops, teachers

- learn how to judge the quality of existing assessment instruments and to select instruments appropriate for their instructional goals,
- reflect on the goals and nature of formative assessment in light of desired student learning outcomes,
- work in detail on “scoring” tasks, and
- examine how assessment task items are constructed to reflect hypothetical learning and assessment trajectories.

“We found that teachers’ overall instructional practice became more flexible,” says Romberg, “both to the class as a whole and to the individual student, in addition to being more sensitive to students’ understanding of mathematics.” An increased attention to student learning, explicit expectations of student performance, and stronger individual and group feedback helped students progress from informal to formal reasoning in mathematical domains. Using formative evaluation tools gives teachers an early impression of student misconceptions and provides more time for teachers to adjust their instructional plans.

“Twenty years of developmental research suggests that it is unrealistic to expect teachers to become instant assessment designers and experts.”

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Physics teacher is making a presentation to the class. A student asks, “Should we write any of that down?” and the teacher responds, “Sure.”

Did the student’s question make a good impression or a bad impression on the teacher?

Each time students participate in class they leave themselves open for an assessment by the teacher, positive or negative. These incremental assessments have implications for the students’ class grades as well as their future options. But while classroom discussion may involve an element of risk, an important part of learning occurs here.

Zuengler and colleagues at WCER’s Center on English Learning and Achievement (CELA) are examining learning by students who are English language learners, compared with those who are monolingual in English. Their research is funded by the Office of Educational Research and Improvement, U.S. Department of Education. The team’s recent findings have implications for the teaching and subject matter learning of English language learners. Many students need to be guided in the ways of participating in mainstream subject matter classroom talk. How a student initiates an interaction is important.

However, Zuengler and colleagues have not observed subject matter teachers offering students substantial guidance in the process of talk in their classes.

Zuengler’s team observes classrooms in a high school in a midwestern city—the most linguistically and ethnically diverse high school in its urban area. Sixty percent of the students are Hispanic, 20 percent are African American, 10 percent are White, 8 percent are Asian, and 2 percent are Native American. The team is documenting the socialization to subject matter language of a cohort of students that entered the school as ninth graders in fall 1997. The study has followed this cohort in their science and social studies classrooms through their junior year.

Depending on their language proficiency, second-language learners take either one or two classes in the school’s ESL program. This program serves students whose native languages include Spanish, Hmong, Lao, Thai, and Arabic. The research team videotapes classroom activities and focuses on the “risky” moments when students initiate classroom interaction. In considering the teacher’s response to each of the student initiations, some utterances proved less risky than others for the student producing them.

In a physics class of juniors and seniors, Mr. James, the teacher, has been standing up front explaining the transfer of energy from one form to another. (All names are pseudonyms.) In his presentation, Mr. James points to text and diagrams on the overhead projector, plays short clips on the VCR, and demonstrates using his hands and arms. In this excerpt the teacher is looking and pointing to the overhead.

A student, Ramon, asks, “Should we write any of that down?” and Mr. James responds, “Sure.” The outcome of this exchange is that Ramon and several other students in the class, none of whom had been taking notes up until this point, got out their notebooks and began copying down the information. In so doing, Ramon and the others displayed their willingness to follow the advice given and their intent to take down the material that Mr. James had just emphasized.

One might say then that Ramon’s question and the teacher’s ratification of that led him and several others toward efforts that at least corresponded to what Mr. James thought important enough for them to learn. However, in a conversation with the teacher after class, researchers found that he condemned Ramon’s question. Complaining about the class as a whole, Mr. James shook his head as he described a class of what he called “lazy” students who “just think they can pick up everything by sitting there, not doing anything of their own initia-
tive, without keeping up with any of the reading
outside of class." Portraying his physics students as
cluless, Mr. James brought up Ramon’s question
and used it to make the point that what Ramon
had asked just confirmed what Mr. James had been
complaining about.

Zuengler says this example represents interac-
tional patterns that are frequent. There is a place in
the flow of classroom talk where the student has
initiated some exchange and put himself on dis-
play. Ramon’s placing his question on the floor led
to a confirming negative assessment by the
teacher.

"From our research," Zuengler says, "it appears
that when students draw on subject matter, embed
their comment or question in some larger context,
or offer an account for why they are raising a par-
ticular observation or question, they are far more
successful than when the more ‘bald-faced’ com-
ments are placed in the conversational space." For
example, another student, Jennifer, questions the
teacher as they discuss exponents in scientific
notation.

Mr. Walsh: Ten to the eleventh. . . . notice I just
add the five and the six. . . .

Jennifer: But that contradicts what were doing in
Algebra right now with, um, the order of opera-
tions. . . .

Mr. Walsh: Okay. What here goes against the
order of operations?

Jennifer: (I don't know) I understand what you're
doing like that but . . . . That goes against the order
of operations. . . .

Mr. Walsh: Okay. . . . Ten to the sixth is the same as
ten times ten times ten times ten times ten times
ten. Six of them. It's just easier to write it this way. . .

Jennifer’s question contains some elements of
“challenge” to the teacher, but she says “but that
contradicts . . . “ and “that goes against . . . “ so Mr.
Walsh accepts it as something to pursue in his talk.

This difference in the teacher’s reactions, says
Zuengler, may be that teachers and students are all
doing many complex and interwoven activities in
any given class period. This complexity means that
for any student utterance there are many possible
interpretations of the student’s intent and the
“question behind the question.” In the absence of
any other contextualization, the teacher freely
interprets, or must build a context within which to
interpret, the student’s comment. With any luck,
the intent and interpretation match, and the
teacher responds to the real question that is asked,
whether it is the one that is uttered or not.

However, it is possible for the teacher to inter-
pret the student’s question in ways that put the stu-
dent at risk for a negative assessment. By providing
an account or framing the question, the student
has the chance to control the teacher’s interpreta-
tion of the turn. This control may also help shape
the teacher’s assessment of the student or class by
students taking an active role in shaping the posi-
tions and identities that are ascribed to them.

"In our experience talking with both students
and teachers about successful communication," says
Zuengler, "the ways by which students initiate
classroom exchanges never comes up; this suggests
that it’s probably below their consciousness. If this
is true, then it raises some important implications
for what ESL teachers might do with their students
to reduce the conversational risks they encounter
in their mainstream classes."

Implications

How a student initiates an interaction is important;
however, other than socializing students to be
quiet and not swear, Zuengler and colleagues have
not seen or heard subject matter teachers offering
any guidance in how to talk, on the process of talk,
in their classes. Instead, they seem to make the
assumption that students already know, or should
know, how to talk—by having learned it before
coming into their class. Yet many examples show
that that assumption is unfounded.

While it may not be true for subject matter
classes, within the ESL classes the ways of talking,
the process of talk, are emphasized. Since the ESL
teachers are guiding learners in their acquisition of
English, and since their learners are often already
taking some mainstream classes, Zuengler would
encourage ESL teachers to consider some ways in
which their learners can reduce risks in their main-
stream classroom participation.

Zuengler and colleagues recommend the fol-
lowing as possible actions for ESL teachers to take:
1. Determine what the talk is like in your stu-
dents’ subject matter classes.
2. Begin conversations with subject-matter teach-
ers about their beliefs and assumptions about
appropriate student talk, and how they believe
students learn to talk in their classrooms.
3. Train students to do contextualizing work tai-
lored to those beliefs and assumptions.

For more information visit the CELA web site
at www.wcer.wisc.edu/cela or contact Zuengler at
zuengler@facstaff.wisc.edu.
Writing effective arguments

Writing well is not easy. Writing an effective argument provides an even bigger challenge.

Writing an effective argumentative essay means getting your own points across while anticipating and addressing the reactions of a potential reader. But lacking proper guidance, students assigned to compose argumentative papers often end up writing reports, rather than arguments.

UW–Madison English professor Martin Nystrand and colleagues at WCER’s Center for English Learning and Achievement (CELA) found that teaching students to construct effective arguments lies not just in the narrow confines of writing instruction, but in the larger classroom context. Their work is funded by the Office of Educational Research and Improvement, U.S. Department of Education.

In writing argumentation, student writers need to know how to put themselves in the position of the reader. They need to know how to question and disagree with the points being made in the argument. Yet these processes are often short-circuited when the teacher routinely treats knowledge as a given—fixed and found in texts.

Nystrand and colleague Nelson Graff examined an excellent writing teacher, Mrs. Martin (a pseudonym), whose seventh-grade students were having difficulty producing effective written arguments. Nystrand says that, in spite of the best efforts of this skilled and dedicated teacher, it was a challenging and not fully successful effort. The fact that her students still weren’t writing successful arguments despite her up-to-date professionalism became the research focus.

The students wrote “hybrid” papers: Their essays included argumentative theses, but were developed with loosely related “factoids,” rather than with argumentative points that built upon each other to form a persuasive point of view—even though Mrs. Martin had required students to go through cognitive strategies for argumentative analysis in classroom ‘think-aloud’ exercises.

For example, a student named Carmen composed a paper titled “African Elephants Should Not Be Killed.” Her thesis was, “They [elephants] should be rescued and treated fairly.” Her conclusion read, “In over one-hundred years elephants have not been treated fairly, at all.” In between, we mainly learn about elephants: “Elephants are endangered because they keep getting killed for their ivory tusks, meat, and skin.” “Resources are made out of elephants such as: ivory pool balls, jewelry, leather, table tops, shoes, belts, boots, piano keys, combs, brushes, and more.” “Elephants are valuable resources in National Parks,” and “Even though killing an elephant is illegal and poachers still do it, elephant researchers may need to take the elephants into captivity, raise them for a few years and put them back into the wild to keep the population up.”

Previous studies had documented important parallels and relations between the development of writing and reading, yet most research had tended to focus on either writing or reading, not the two in interaction. Nystrand saw the value of situating writing development amidst reading and classroom discourse and other class activities. He then focused on classroom discourse and the role it plays in shaping student learning.
Mrs. Martin practiced many commendable techniques: She responded to students’ drafts and their final copies. She conducted regular writing conferences, made effective use of small groups, and taught a diverse range of assignments, from ungraded journals to argumentative prose. “On any measures of professional awareness and commitment to the standards of process instruction, the teacher surely scored in the top percentiles,” Nystrand says. “Yet her students’ efforts at argumentation persistently stalled as they fell back on mastered skills in reporting given information rather than configuring it to effective rhetorical use.”

In this case the teacher had neglected to teach the students how to think through the process of constructing an argument. She invited students to ask questions about research procedures, but student questions never led to discussion. Like most language arts classes in middle school, the teacher talked and the students listened. And although Martin asked her students to write arguments that came from making their own interpretations of the materials they collected, she discussed writing merely in terms of form, and her students mainly learned argument as a matter of text elaboration—information and details, not always related to claims.

Observations of this teacher’s class revealed that effectively teaching rhetoric and argumentation means more than adopting innovative strategies. It means modifying one’s ideas about knowledge and writing and examining all the activities of the classroom for the resources they offer students for developing rhetorical skills.

A few changes in writing instruction, while important, may not have the desired results if the dominant teaching mode of the classroom derails the instructional goals for writing. In this teacher’s case, comments on papers and during classroom discussion tended to close discourse, rather than to extend it and to encourage exploration and learning.

The classroom tended to favor efficient recitation, recall, and a mastery of givens, rather than vigorous discussion and argument. “But when writers construct arguments and move through the process of composing texts, they transcend the givens,” Nystrand says. “They don’t just find and report knowledge piecemeal. Students need to be shown how to weave together information supported by their developing understanding.”

Writing teachers should give students opportunities to figure things out—in class, face-to-face, teacher and students together. Students’ tasks should go beyond recalling “precast” knowledge to tasks in which they are dynamically involved in constructing new understandings in classroom interaction.

Nystrand and Graff hope that this study helps even the most experienced teachers of writing think productively about their teaching.

For more information see the CELA web site at www.wcer.wisc.edu/CELA or contact Nystrand at nystrand@ssc.wisc.edu.
Classroom assessment

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deLange says. “But classroom assessment can serve as a basis for reorienting teacher practice so that it is flexible and more sensitive to students’ understanding of mathematics.”

Formative assessment activities are often most effective when they permit teacher-student interaction and probing of responses. This interactive approach enables the teacher to assess individual students’ informal or formal knowledge and to repeat important topics when needed. Although basic knowledge and skills can be evaluated in an initial more formal assessment, the informal assessment permits teachers to check higher-level student competencies such as non-routine problem solving and mathematical communication.

As a result of professional development in the use of formative assessments, teachers alter the use and choice of assessment instruments, the level of addressing learning for understanding, continuity of the assessment process, feedback to students, instructional practices, methods of scoring assessments and assigning grades, and perspectives in the ways their students learn.

Researchers are collecting data and will produce more reports contributing to a theory of classroom assessment, Romberg says. “We hope this work will help teachers document how individual students progress from informal to formal understanding of concepts and procedures in both mathematics and science.”

Two findings from this research stand out, says deLange. First, most mathematics teachers have limited understanding of formative assessment practices, and they consequently provide students with incomplete information about their progress. Second, teachers can learn to use such practices in their classrooms as a consequence of appropriate professional development. In turn, their students’ achievement improves.

For more information visit www.wcer.wisc.edu/NCISLA.